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Relating the Green-Schwarz and pure spinor formalisms for the superstring

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Abstract: Although it is not known how to covariantly quantize the Green-Schwarz (GS) superstring, there exists a semi-light-cone gauge choice in which the GS superstring can be quantized in a conformally invariant manner. In this paper, we prove that BRST quantization of the GS superstring in semi-light-cone gauge is equivalent to BRST quantization using the pure spinor formalism for the superstring

Keywords: Superstrings and Heterotic Strings, BRST Symmetry.

Contents

1. Introduction

Four years ago, a new manifestly super-Poincaré covariant formalism was introduced for quantizing the superstring [1]. This formalism has been recently used for computing covariant multiloop amplitudes [2] and for quantization in an $AdS_5 \times S^5$ Ramond-Ramond background [3]. The main new ingredient in this formalism is a BRST operator constructed from fermionic Green-Schwarz (GS) constraints and a pure spinor bosonic ghost.

At the present time, a geometrical understanding of this "pure spinor" BRST operator is still lacking.[1] It is therefore important to try to relate the pure spinor BRST operator with BRST operators which appear in other formalisms for the superstring. In reference [6], this was done for the RNS formalism where the pure spinor BRST operator was related to the sum of the RNS BRST operator and η ghost. It was also shown in [7] that the cohomology of the pure spinor BRST operator reproduces the light-cone GS spectrum.[2]

Although it is not known how to covariantly quantize the GS superstring, one can choose a semi-light-cone gauge in which κ-symmetry is fixed but conformal invariance is preserved [13]. The worldsheet action is quadratic in semi-light-cone gauge, so quantization is straightforward using the BRST method. Although Lorentz invariance is not manifest in this gauge, one can construct Lorentz generators whose algebra closes up to a BRST-trivial quantity.

[1]However, there are some indications that the pure spinor BRST operator can be interpreted as a twisted N=2 worldsheet supersymmetry generator using either a "twistor-superstring" formalism [4] or a WZNW model [5].

[2]There also exist "extended" versions of the pure spinor formalism in which the pure spinor constraint on the bosonic ghost is relaxed [8, 9]. The BRST operator in these extended pure spinor formalisms have been related to the RNS BRST operator [10], to the light-cone GS spectrum [11], and to the original pure spinor BRST operator [12].

In this paper, the pure spinor BRST operator will be related to the semi-light-cone GS BRST operator by a similarity transformation, proving the equivalence of the cohomologies. Note that a similar result was obtained earlier for the $d = 10$ [14] and $d = 11$ superparticle [15] in semi-light-cone gauge. However, relating the BRST operators for the superstring is more complicated than for the superparticle because of normal-ordering subtleties. It would be interesting to try to further generalize this equivalence proof to the $d = 11$ supermembrane for which pure spinor [16] and semi-light-cone gauge descriptions exist at least at the classical level.

In section 2 of this paper, we review the equivalence proof for the $d = 10$ superparticle. And in section 3, we generalize this equivalence proof to the superstring.

2. Review of equivalence proof for superparticle

2.1 Brink-Schwarz superparticle in the semi-light-cone gauge

The $d = 10$ Brink-Schwarz superparticle is described by the action

$$S = \int d\tau \left(\dot{x}^m P_m - \frac{i}{2}(\dot{\theta}\gamma^m\theta)P_m + eP^m P_m \right) , \tag{2.1}$$

where P_m is the conjugate momentum for x^m, θ^α is a spinor of SO(9,1), e is a Lagrange multiplier which enforces the mass-shell condition, $\gamma^m_{\alpha\beta}$ are 16×16 symmetric gamma matrices which satisfy $\gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\lambda} = 2\eta^{mn}\delta^\lambda_\alpha$, and we use the metric convention $\eta^{mn} = \mathrm{diag}(-1, 1, 1, \ldots)$.

The action (2.1) is spacetime supersymmetric and is also invariant under local kappa transformations, which are generated by the first-class part of the fermionic constraints

$$d_\alpha = p_\alpha + 2P^m(\gamma_m\theta)_\alpha , \tag{2.2}$$

which satisfy the Poisson brackets

$$\{d_\alpha, d_\beta\} = 4P_m\gamma^m_{\alpha\beta} . \tag{2.3}$$

Eight of the d_α constraints are first-class and the other eight are second-class. There is no simple way of covariantly separate out the first-class constraints, preventing covariant quantization. Nevertheless, it is possible to use a non-Lorentz covariant gauge fixing to quantize this theory by imposing the condition $(\gamma^+\theta)_\alpha = 0$ with the assumption that $P^+ \neq 0$ (where $\gamma^\pm = \gamma^0 \pm \gamma^9$ and $P^\pm = P^0 \pm P^9$). In this semi-light-cone gauge, the action takes the form

$$S = \int d\tau \left(\dot{x}^m P_m + \frac{i}{2}\dot{S}_a S_a + eP^m P_m \right) , \tag{2.4}$$

where $S_a = \sqrt{\frac{P^+}{2}}(\gamma^-\theta)_a$ and $a = 1$ to 8 is an SO(8) chiral spinor index. Canonical quantization of (2.4) implies that $\{S_a, S_b\} = \delta_{ab}$ and therefore $\sqrt{2}S_a$ acts like a spinor version of SO(8) Pauli matrices that satisfy

$$\sigma^j_{a\dot{a}}\sigma^j_{b\dot{b}} + \sigma^j_{b\dot{a}}\sigma^j_{a\dot{b}} = 2\delta_{ab}\delta_{\dot{a}\dot{b}} . \tag{2.5}$$

Making use of the usual BRST method, the action (2.4) can be gauge-fixed to

$$S = \int d\tau \left(\dot{x}^m P_m + \frac{i}{2}\dot{S}_a S_a - \frac{1}{2}P^m P_m + i\dot{c}b \right), \tag{2.6}$$

where the BRST charge is

$$Q = cP^m P_m \,. \tag{2.7}$$

Note that κ-symmetry ghosts do not propagate in semi-light-cone gauge, so the BRST operator only involves the (b, c) reparametrization ghosts.

Because semi-light-cone gauge is not manifestly Lorentz invariant, Lorentz transformations which change the gauge-fixing condition $(\gamma^+\theta)_\alpha = 0$ need to include a compensating κ-transformation. The resulting Lorentz generators are

$$\begin{aligned} N^{+-} &= -ix^+P^- + ix^-P^+ \,, \\ N^{i+} &= -ix^iP^+ + ix^+P^i \,, \\ N^{i-} &= -ix^iP^- + ix^-P^i - \frac{(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}P^j}{2P^+} \,, \\ N^{ij} &= -ix^iP^j + ix^jP^i - \frac{1}{4}(S_a\sigma^{ij}_{ab}S_b) \,, \end{aligned} \tag{2.8}$$

where $x^\pm$, $P^\pm$ are light-cone variables, and $i, j = 1$ to 8. Using the canonical commutation relations $\{S_a, S_b\} = \delta_{ab}$ and $[P^m, x^n] = -i\eta^{mn}$, one finds that the Lorentz generators of (2.8) satisfy the usual $\mathrm{SO}(9, 1)$ Lorentz algebra except for $[N^{i-}, N^{j-}]$, which satisfies

$$[N^{i-}, N^{j-}] = \left[Q, -\frac{b(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}}{(P^+)^2} \right]. \tag{2.9}$$

Since $[N^{i-}, N^{j-}]$ is BRST-trivial, the Lorentz algebra closes up to a gauge transformation when acting on BRST-invariant states. In other words, $QV = 0$ implies that $[N^{i-}, N^{j-}]V = Q\Omega$ for some Ω.

2.2 Equivalence between Brink-Schwarz superparticle and pure spinor superparticle

In this subsection, it will be shown that the action and BRST operator for the semi-light-cone Brink-Schwarz superparticle in the previous subsection are related to the action

$$S = \int d\tau \left(\dot{x}^m P_m - \frac{1}{2}P^m P_m + i\dot{\theta}^\alpha p_\alpha + i\dot{\lambda}^\alpha w_\alpha \right), \tag{2.10}$$

and BRST operator

$$Q = \lambda^\alpha d_\alpha \tag{2.11}$$

for the pure spinor version of the superparticle [17] where $(\lambda^\alpha, w_\alpha)$ are bosonic ghosts satisfying the pure spinor constraint

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0 \,. \tag{2.12}$$

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To relate the actions of (2.6) and (2.10), we shall first introduce a new pair of fermionic variables $(\theta^\alpha, p_\alpha)$ which are not related with S_a. And to prevent physical states from depending on these new variables, we shall also introduce a new gauge invariance which allows θ^α to be gauged to zero. This new gauge invariance will be generated by the first-class constraints

$$\widehat{d}_\alpha = d_\alpha + \frac{(\gamma_m \gamma^+ S)_\alpha P^m}{\sqrt{P^+}}\,, \tag{2.13}$$

where $d_\alpha = p_\alpha + 2P^m(\gamma_m\theta)_\alpha$. Or using SO(8) notation,

$$\begin{aligned} \widehat{d}_a &= d_a + 2S_a\sqrt{P^+}\,, \\ \widehat{d}_{\dot a} &= d_{\dot a} - \frac{2P^i (S\sigma^i)_{\dot a}}{\sqrt{P^+}}\,, \end{aligned} \tag{2.14}$$

where $a, \dot a = 1$ to 8 are SO(8) chiral and antichiral spinor indices.

Using (2.3) and the anticommutation relation of the S_a's, one can check that the constraints satisfy the first-class algebra

$$\{\widehat{d}_\alpha, \widehat{d}_\beta\} = \frac{2P^m P_m \gamma^+_{\alpha\beta}}{P^+}\,, \tag{2.15}$$

or in SO(8) notation,

$$\{\widehat{d}_a, \widehat{d}_b\} = \{\widehat{d}_a, \widehat{d}_{\dot b}\} = 0\,, \qquad \{\widehat{d}_{\dot a}, \widehat{d}_{\dot b}\} = \frac{4P^m P_m \delta_{\dot a \dot b}}{P^+}\,. \tag{2.16}$$

So the semi-light-cone superparticle action which includes the new variables and new gauge invariances is

$$S = \int d\tau \left(\dot x^m P_m + eP^m P_m + i\dot\theta^\alpha p_\alpha + \frac{i}{2}\dot S_a S_a + f^\alpha \widehat{d}_\alpha \right), \tag{2.17}$$

where f^α are fermionic Lagrange multipliers related to the constraint (2.13). To return to the original action in the semi-light-cone gauge, $\widehat{d}_\alpha$ can be used to gauge $\theta^\alpha = 0$, recovering (2.4). Using the usual BRST method, one can gauge fix the action above and obtain

$$S = \int d\tau \left(\dot x^m P_m - \frac{1}{2}P^m P_m + i\dot\theta^\alpha p_\alpha + \frac{i}{2}\dot S_a S_a + i\dot{\widehat\lambda}^\alpha \widehat w_\alpha + i\dot c b \right), \tag{2.18}$$

together with the BRST charge

$$\widehat{Q} = \widehat\lambda_a \widehat{d}_a + \widehat\lambda_{\dot a}\widehat{d}_{\dot a} + c\left(-4P^- + \frac{4P^i P^i}{P^+} \right) - \frac{1}{2}\widehat\lambda_{\dot a}\widehat\lambda_{\dot a} b\,, \tag{2.19}$$

where $\widehat\lambda^\alpha$ is an unconstrained bosonic spinor ghost related to the gauge fixing $f^\alpha = 0$, $\widehat w_\alpha$ is its conjugate momentum, and $a, \dot a, i = 1$ to 8. Note that one could rescale the (b, c) ghosts by factors of P^+ as

$$b \to \frac{b}{P^+}, \quad c \to cP^+\,,$$

so that the c ghost would multiply $P_m P^m$ in the BRST operator. But it will be more convenient for generalization to the superstring to leave the superparticle BRST operator in the form of (2.19). This means that b and c are not invariant under Lorentz transformations which change the value of P^+.

As before, Lorentz invariance is not manifest but one can define Lorentz generators which commute with the BRST operator and whose algebra closes up to a BRST-trivial quantity. The explicit expression for the Lorentz generators is

$$\begin{aligned}
N^{+-} &= -ix^+P^- + ix^-P^+ + \theta_a p_a - \theta_{\dot a} p_{\dot a} + \widehat{\lambda}_a \widehat{w}_a - \widehat{\lambda}_{\dot a}\widehat{w}_{\dot a} - 2bc\,,\\
N^{i+} &= -ix^iP^+ + ix^+P^i - (p\sigma^i\theta) + (\widehat{w}\sigma^i\widehat{\lambda})\,,\\
N^{ij} &= -ix^iP^j + ix^jP^i + \frac{1}{2}(\theta\sigma^{ij}p) + \frac{1}{2}(\theta\overline{\sigma}^{ij}p) + \frac{1}{2}(\widehat{\lambda}\sigma^{ij}\widehat{w}) + \frac{1}{2}(\widehat{\lambda}\overline{\sigma}^{ij}\widehat{w}) - \frac{1}{4}(S\sigma^{ij}S)\,,\\
N^{i-} &= -ix^iP^- + ix^-P^i + \frac{b(S\sigma^i\widehat{\lambda})}{2\sqrt{P^+}} - \frac{(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}P^j}{2P^+} + (\theta\sigma^i p) + (\widehat{\lambda}\sigma^i\widehat{w}) - \frac{2bcP^i}{P^+}\,,
\end{aligned} \tag{2.20}$$

where

$$(A\sigma^iB) = A_a\sigma^i_{a\dot b}B_{\dot b}\,, \qquad (A\sigma^{ij}B) = \frac{1}{2}A_a\sigma^{[i}_{a\dot c}\sigma^{j]}_{b\dot c}B_b\,, \qquad (A\overline{\sigma}^{ij}B) = \frac{1}{2}A_{\dot a}\sigma^{[i}_{c\dot a}\sigma^{j]}_{c\dot b}B_{\dot b}\,.$$

The generators obey the usual Lorentz algebra, except for the commutator of N^{i-} with N^{j-} which satisfies

$$[N^{i-}, N^{j-}] = \left[\widehat{Q}, -\frac{b(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}}{P^+}\right], \tag{2.21}$$

indicating that the algebra closes up to a gauge transformation on on-shell states.

It will now be shown that the BRST operator of (2.19) is related by a similarity transformation to the pure spinor BRST operator $Q = \lambda^\alpha d_\alpha$ [14]. The first step will be to show that the cohomology of the BRST operator in (2.19) is equivalent to the cohomology of $Q' = \widehat{\lambda}_a\widehat{d}_a + \lambda_{\dot a}\widehat{d}_{\dot a}$ in a Hilbert space without the (b, c) ghosts and with the condition $\lambda_{\dot a}\lambda_{\dot a} = 0$.

Suppose that a state V is in the cohomology of Q'. Then V is annihilated by the operator

$$Q'' = \widehat{\lambda}_a\widehat{d}_a + \widehat{\lambda}_{\dot a}\widehat{d}_{\dot a} \tag{2.22}$$

up to terms proportional to $\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}$, i.e.,

$$Q''V = \widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}W\,, \tag{2.23}$$

for some W. Since

$$(Q'')^2 = \frac{2\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}P^mP_m}{P^+}\,, \tag{2.24}$$

(2.23) implies that

$$Q''W = \frac{2P^mP_mV}{P^+}\,. \tag{2.25}$$

which implies that the state $\widehat{V} = V + 2cW$ is annihilated by $\widehat{Q}$. And if V is BRST-trivial up to terms proportional to $\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}$, i.e.,

$$V = Q''\Omega + \widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}Y \tag{2.26}$$

for some Ω and Y, then $\widehat{V} = V + 2cW = \widehat{Q}(\Omega - 2cY)$ is also BRST-trivial.

To prove the converse, i.e. that any state in the cohomology of $\widehat{Q}$ maps to a state in the cohomology of Q', suppose that the state $\widehat{V}$ is in the cohomology of $\widehat{Q}$. By choosing the b ghost to annihilate the vacuum, one can write $\widehat{V} = V + cW$ for some V and W.

Then $\widehat{Q}V = \frac{1}{2}\widehat{\lambda}_{\dot{a}}\widehat{\lambda}_{\dot{a}}W$ implies $Q'V = 0$ in the reduced Hilbert space. And $\widehat{V} = \widehat{Q}\Lambda$ where $\Lambda = \Omega + cY$ implies that $V = \widehat{Q}\Omega - \frac{1}{2}\widehat{\lambda}_{\dot{a}}\widehat{\lambda}_{\dot{a}}Y$, i.e. $V = Q'\Omega$ in the reduced Hilbert space. So the cohomology of Q' is equivalent to the cohomology of $\widehat{Q}$.

Now, it will be shown that the cohomology of $Q' = \widehat{\lambda}_a\widehat{d}_a + \lambda_{\dot{a}}\widehat{d}_{\dot{a}}$ is equivalent to the cohomology of $Q = \lambda^\alpha d_\alpha$ in a Hilbert space independent of S_a, where λ^α is a pure spinor. To do this, it is convenient to define an antichiral spinor $r_{\dot{a}}$ which satisfies $r_{\dot{a}}\lambda_{\dot{a}} = 1$ and $r_{\dot{a}}r_{\dot{a}} = 0$. One can then use $r_{\dot{a}}$ to split the fields S_a and $\widehat{\lambda}_a$ as[3]

$$\begin{aligned} S_a &= S^1_a + S^2_a\,, \\ \widehat{\lambda}_a &= \widehat{\lambda}^1_a + \widehat{\lambda}^2_a\,, \end{aligned} \tag{2.27}$$

where

$$\begin{aligned} S^1_a &= \frac{1}{2}(\sigma^j\lambda)_a(S\sigma^j r)\,, \qquad & S^2_a &= \frac{1}{2}(\sigma^j r)_a(S\sigma^j\lambda)\,, \\ \widehat{\lambda}^1_a &= \frac{1}{2}(\sigma^j\lambda)_a(\widehat{\lambda}\sigma^j r)\,, \qquad & \widehat{\lambda}^2_a &= \frac{1}{2}(\sigma^j r)_a(\widehat{\lambda}\sigma^j\lambda)\,. \end{aligned} \tag{2.28}$$

The new fields have the anticommutation relations

$$\begin{aligned} \{S^1_a, S^2_b\} &= \frac{1}{2}(\sigma^i\lambda)_a(\sigma^i r)_b\,, \\ \{S^1_a, S^1_b\} &= \{S^2_a, S^2_b\} = 0\,. \end{aligned} \tag{2.29}$$

And the charge Q', written in terms of the new fields, reads

$$Q' = \widehat{\lambda}^1_a d_a + \widehat{\lambda}^2_a d_a + 2\widehat{\lambda}^1_a S^2_a\sqrt{P^+} + 2\widehat{\lambda}^2_a S^1_a\sqrt{P^+} + \lambda_{\dot{a}}d_{\dot{a}} - \frac{2(S^2\sigma^i\lambda)P^i}{\sqrt{P^+}}\,. \tag{2.30}$$

Performing the similarity transformation

$$\begin{aligned} Q' &\longrightarrow e^{-\frac{d_a S^2_a}{2\sqrt{P^+}}}Q'e^{+\frac{d_a S^2_a}{2\sqrt{P^+}}} \\ &= \lambda_{\dot{a}}d_{\dot{a}} + \widehat{\lambda}^1_a d_a + 2\widehat{\lambda}^2_a S^1_a\sqrt{P^+}\,, \end{aligned} \tag{2.31}$$

one obtains $Q' = \lambda^\alpha d_\alpha + 2\widehat{\lambda}^2_a S^1_a\sqrt{P^+}$, where λ^α is a pure spinor defined by

$$[\lambda_{\dot{a}}, \lambda_a] = [\lambda_{\dot{a}}, \widehat{\lambda}^1_a]\,. \tag{2.32}$$

Using the quartet argument, the cohomology of $Q' = Q + 2\widehat{\lambda}^2_a S^1_a\sqrt{P^+}$ is equivalent to the cohomology of $Q = \lambda^\alpha d_\alpha$ in the Hilbert space independent of $\widehat{\lambda}^2_a$ and S^1_a, and independent of their conjugate momenta $\widehat{w}^1_a$ and S^2_a. Therefore, the action and BRST operator for the Brink-Schwarz superparticle in semi-light-cone gauge are equivalent to the action

$$S = \int d\tau\left(\dot{x}^m P_m - \frac{1}{2}P^m P_m + i\dot{\theta}^\alpha p_\alpha + i\dot{\lambda}^\alpha w_\alpha\right), \tag{2.33}$$

and the BRST operator $Q = \lambda^\alpha d_\alpha$ where $(\lambda\gamma^m\lambda) = 0$.

[3]In order that the cohomology remain non-trivial after including $r_{\dot{a}}$, states will only be allowed to depend on $r_{\dot{a}}$ in the combination $(\sigma^j r)_a(\sigma^j\lambda)_b$. If states could depend arbitrarily on $r_{\dot{a}}$, the cohomology would become trivial since $QV = 0$ implies that $Q(\theta^{\dot{a}}r_{\dot{a}}V) = V$.

3. Equivalence proof for Green-Schwarz superstring

3.1 Green-Schwarz superstring in the semi-light-cone gauge

As in the covariant Brink-Schwarz superparticle action, the GS superstring action contains first-class and second-class constraints which are difficult to separate in a covariant manner. Quantization can be performed in a conformally invariant manner by gauge-fixing κ-symmetry using the condition $(\gamma^+\theta)_\alpha = 0$, assuming that $\partial X^+ \neq 0$. In this semi-light-cone gauge, the GS action is written as

$$S = \frac{1}{\pi}\int d^2z\left[\frac{1}{2}\partial X^m\overline{\partial}X_m + \frac{1}{2}S_a\overline{\partial}S_a + \text{anti-holomorphic terms}\right], \tag{3.1}$$

where $S_a = \sqrt{\frac{\partial X^+}{2}}(\gamma^-\theta)_a$ is a chiral SO(8) spinor. The anti-holomorphic terms in (3.1) depend if one is discussing the Type II or heterotic superstring, and will be ignored in this paper.

In semi-light-cone gauge, one can construct a BRST charge in the standard manner as

$$Q = \int dz(cT_m + bc\partial c) \tag{3.2}$$

with the action

$$S = \frac{1}{\pi}\int d^2z\left[\frac{1}{2}\partial X^m\overline{\partial}X_m + \frac{1}{2}S_a\overline{\partial}S_a + b\overline{\partial}c + \text{anti-holomorphic terms}\right], \tag{3.3}$$

where

$$T_m = -\partial X^-\partial X^+ + \partial X^i\partial X^i - \frac{1}{2}S_a\partial S_a + \frac{1}{2}\partial^2(\log\partial X^+) \tag{3.4}$$

is the stress tensor. The term $\frac{1}{2}\partial^2(\log\partial X^+)$ in the stress tensor comes from the non-covariant gauge-fixing and, as will be shown below, is necessary both for quantum conformal invariance and quantum Lorentz invariance.[4] Using the OPE's

$$\begin{aligned} X^m(y,\overline{y})X^n(z,\overline{z}) &\longrightarrow \frac{1}{2}\eta^{mn}\log\mid y-z\mid^2, \\ S_a(y)S_b(z) &\longrightarrow \frac{\delta_{ab}}{y-z}, \end{aligned} \tag{3.5}$$

one finds that T_m has central charge $c = 26$, so Q is nilpotent at the quantum level.

Although Lorentz invariance is not manifest, one can construct Lorentz generators that commute with Q. The holomorphic components of the currents for these generators are

$$\begin{aligned} N^{ij} &= -X^i\partial X^j + X^j\partial X^i - \frac{1}{4}(S\sigma^{ij}S), \\ N^{+-} &= -\frac{1}{2}X^+\partial X^- + \frac{1}{2}X^-\partial X^+, \\ N^{i+} &= -X^i\partial X^+ + X^+\partial X^i, \\ N^{i-} &= -X^i\partial X^- + X^-\partial X^i - \frac{(S\sigma^i)_{\dot{a}}(S\sigma^j)_{\dot{a}}\partial X^j}{2\partial X^+}. \end{aligned} \tag{3.6}$$

[4] Although there are many papers which discuss anomalies in semi-light-cone-gauge for the GS superstring [18], we are not aware of any discussion which uses this BRST method.

As in the superparticle case, the algebra closes up to a BRST-trivial operator:

$$\left[\int dy N^{i-}(y), \int dz N^{j-}(z)\right] = \left[Q, \int dz\left[-\frac{b(S\sigma^i)_{\dot{a}}(S\sigma^j)_{\dot{a}}}{(\partial X^+)^2}\right](z)\right]. \tag{3.7}$$

So after including the term $\frac{1}{2}\partial^2(\log\partial X^+)$ in T_m, the Lorentz algebra closes on on-shell states up to a gauge transformation.

3.2 Equivalence between Green-Schwarz and pure spinor formalisms

As with the superparticle, we shall add a new pair of fermionic degrees of freedom $(\theta^\alpha, p_\alpha)$ not related to S_a, and a new gauge invariance which allows θ^α to be gauged to zero. The new gauge invariance will be generated by sixteen first-class constraints $\widehat{d}_\alpha$ constructed from the S_a variables and the first and second-class GS constraints d_α. For the GS superstring, the holomorphic first and second-class constraints are

$$d_\alpha = p_\alpha + (\theta\gamma^m)_\alpha(2\partial X_m + (\theta\gamma_m\partial\theta)), \tag{3.8}$$

which satisfy the OPE's

$$d_\alpha(y)d_\beta(z) \longrightarrow \frac{4\gamma^m_{\alpha\beta}\Pi_m}{y-z}, \tag{3.9}$$

where $\Pi^m = \partial X^m - (\theta\gamma^m\partial\theta)$.

By combining these constraints with the S_a variables satisfying the OPE's

$$S_a(y)S_b(z) \longrightarrow \frac{\delta_{ab}}{y-z}, \tag{3.10}$$

one can construct the sixteen first-class constraints

$$\begin{aligned}\widehat{d}_a &= d_a + 2S_a\sqrt{\Pi^+},\\ \widehat{d}_{\dot{a}} &= d_{\dot{a}} - \frac{2\Pi^i(S\sigma^i)_{\dot{a}}}{\sqrt{\Pi^+}} + \frac{(S\sigma^{jk}S)(\overline{\sigma}^{jk}\partial\theta)_{\dot{a}}}{4\Pi^+} - \frac{4\partial^2\theta_{\dot{a}}}{\Pi^+} + \frac{2\partial\Pi^+\partial\theta_{\dot{a}}}{(\Pi^+)^2},\end{aligned} \tag{3.11}$$

where $a, \dot{a} = 1$ to 8. The constraints of (3.11) obey the first-class algebra:

$$\begin{aligned}\widehat{d}_{\dot{a}}(y)\widehat{d}_{\dot{b}}(z) &\longrightarrow \frac{\widetilde{T}(z)\delta_{\dot{a}\dot{b}}}{y-z},\\ \widehat{d}_{\dot{a}}(y)\widehat{d}_b(z) &\longrightarrow \text{regular},\\ \widehat{d}_a(y)\widehat{d}_b(z) &\longrightarrow \text{regular},\end{aligned} \tag{3.12}$$

where

$$\begin{aligned}\widetilde{T} = &-4\Pi^- + \frac{8S_a\partial\theta_a}{\sqrt{\Pi^+}} - \frac{8\Pi^i(S\sigma^i\partial\theta)}{(\Pi^+)^{3/2}} - \frac{2S_a\partial S_a}{\Pi^+} + \frac{4\Pi^i\Pi^i}{\Pi^+} +\\ &+ \frac{4(S\sigma^i\partial\theta)(S\sigma^i\partial\theta)}{(\Pi^+)^2} - \frac{16\partial^2\theta_{\dot{c}}\partial\theta_{\dot{c}}}{(\Pi^+)^2} - \frac{2\partial^2(\log\Pi^+)}{\Pi^+}.\end{aligned} \tag{3.13}$$

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$\widetilde{T}$ is also a first-class quantity which satisfies the OPE's

$$
\begin{aligned}
\widetilde{T}(y)\widetilde{T}(z) &\longrightarrow \text{regular}\,,\\
\widetilde{T}(y)\,\widehat{d}_{\dot a}(z) &\longrightarrow \text{regular}\,,\\
\widetilde{T}(y)\,\widehat{d}_{a}(z) &\longrightarrow \text{regular}\,.
\end{aligned} \tag{3.14}
$$

The following OPE's were used in these calculations:

$$
\begin{aligned}
d_a(y)d_b(z) &\longrightarrow -\frac{4\delta_{ab}\Pi^+}{y-z}\,,\\
d_a(y)d_{\dot a}(z) &\longrightarrow \frac{4\sigma^i_{a\dot a}\Pi^i}{y-z}\,,\\
d_{\dot a}(y)d_{\dot b}(z) &\longrightarrow -\frac{4\delta_{\dot a\dot b}\Pi^-}{y-z}\,,\\
d_a(y)\Pi^+(z) &\longrightarrow \text{regular}\,,\\
d_a(y)\Pi^-(z) &\longrightarrow \frac{4\partial\theta_a}{y-z}\,,\\
d_a(y)\Pi^i(z) &\longrightarrow \frac{2(\sigma^i\partial\theta)_a}{y-z}\,,\\
d_{\dot a}(y)\Pi^+(z) &\longrightarrow \frac{4\partial\theta_{\dot a}}{y-z}\,,\\
d_{\dot a}(y)\Pi^-(z) &\longrightarrow \text{regular}\,,\\
d_{\dot a}(y)\Pi^i(z) &\longrightarrow \frac{2(\sigma^i\partial\theta)_{\dot a}}{y-z}\,.
\end{aligned} \tag{3.15}
$$

Using the first-class constraints $\widehat{d}_\alpha$ and $\widetilde{T}$, one can construct a nilpotent BRST operator in the usual manner as

$$
\widehat{Q} = \int dz(c\widetilde{T} + \widehat{\lambda}_{\dot a}\widehat{d}_{\dot a} + \widehat{\lambda}_a\widehat{d}_a - \frac{1}{2}\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}b)\,, \tag{3.16}
$$

with the worldsheet action

$$
S = \frac{1}{\pi}\int d^2z\left[\frac{1}{2}\partial X^m\overline{\partial}X_m + \frac{1}{2}S_a\overline{\partial}S_a + p_\alpha\overline{\partial}\theta^\alpha + \widehat{w}_\alpha\overline{\partial}\widehat{\lambda}^\alpha + b\overline{\partial}c\right], \tag{3.17}
$$

where (b,c) are the fermionic ghosts for the bosonic constraint $\widetilde{T}$, and $(\widehat{\lambda}^\alpha, \widehat{w}_\alpha)$ are unconstrained bosonic spinorial ghosts.

As in the superparticle, one could rescale $b \to \frac{b}{\Pi^+}$ and $c \to c\Pi^+$ so that c multiplies the standard stress tensor in the BRST operator. This can be done at the quantum level using the similarity transformation

$$
Q \to e^{-\int dz bc\log(\Pi^+)}Qe^{+\int dz bc\log(\Pi^+)} = 4\int dz(cT_m + bc\partial c + \ldots)\,, \tag{3.18}
$$

where $\int dz(cT_m + bc\partial c)$ is the BRST operator of (3.3) and $\ldots$ involves the new variables $(\theta^\alpha, p_\alpha)$ and $(\widehat{\lambda}^\alpha, \widehat{w}_\alpha)$. However, it will be more convenient to not rescale the (b,c) ghosts so that $\widehat{Q}$ has the simple structure of (3.16). The usual stress tensor can be obtained from

$\widehat{Q}$ by

$$T = \{\widehat{Q}, \frac{1}{4} b\Pi^+ - \widehat{w}_\alpha \partial\theta^\alpha\}$$
$$= \Pi^m\Pi_m - \frac{1}{2} S_a\partial S_a - d_\alpha\partial\theta^\alpha - \widehat{w}_\alpha\partial\widehat{\lambda}^\alpha - b\partial c - \frac{1}{2}\partial^2(\log\Pi^+)\,, \tag{3.19}$$

so $\frac{1}{4}b\Pi^+ - \widehat{w}_\alpha\partial\theta^\alpha$ plays the role of the usual b ghost. Using the OPE's of (3.5) and (3.15), one can verify that T has no central charge. Note that the (b,c) ghosts in (3.19) have not been rescaled so they carry conformal weight $(1,0)$ instead of $(2,-1)$.

As before, Lorentz invariance is not manifest but Lorentz generators can be constructed which leave $\widehat{Q}$ invariant. The holomorphic components of the currents for these generators are

$$N^{ij} = -X^i\partial X^j + X^j\partial X^i + \frac{1}{2}(\widehat{\lambda}\sigma^{ij}\widehat{w}) + \frac{1}{2}(\widehat{\lambda}\overline{\sigma}^{ij}\widehat{w}) + \frac{1}{2}(\theta\sigma^{ij}p) + \frac{1}{2}(\theta\overline{\sigma}^{ij}p) - \frac{1}{4}(S\sigma^{ij}S)\,,$$
$$N^{+-} = -\frac{1}{2}X^+\partial X^- + \frac{1}{2}X^-\partial X^+ + \frac{1}{2}\widehat{\lambda}_a\widehat{w}_a - \frac{1}{2}\widehat{\lambda}_{\dot a}\widehat{w}_{\dot a} + \frac{1}{2}\theta_a p_a - \frac{1}{2}\theta_{\dot a}p_{\dot a} - bc\,,$$
$$N^{i+} = -X^i\partial X^+ + X^+\partial X^i + (\widehat{w}\sigma^i\widehat{\lambda}) - (p\sigma^i\theta)\,,$$
$$N^{i-} = -X^i\partial X^- + X^-\partial X^i - \frac{(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}\Pi^j}{2\Pi^+} + \frac{(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}(S\sigma^j\partial\theta)}{3(\Pi^+)^{3/2}} + \frac{2(\partial S\sigma^i\partial\theta)}{(\Pi^+)^{3/2}} -$$
$$- \frac{2(S\sigma^i\partial\theta)\partial\Pi^+}{(\Pi^+)^{5/2}} + \frac{b(S\sigma^i\widehat{\lambda})}{2\sqrt{\Pi^+}} + \frac{2bc(S\sigma^i\partial\theta)}{(\Pi^+)^{3/2}} - \frac{2bc\Pi^i}{\Pi^+} + (\widehat{\lambda}\sigma^i\widehat{w}) + (\theta\sigma^i p)\,. \tag{3.20}$$

Once again, they obey the usual Lorentz algebra except for N^{i-} with N^{j-}, which satisfies

$$\left[\int dy N^{i-}(y), \int dz N^{j-}(z)\right] = \left[\widehat{Q}, \int dz \left[-\frac{b(S\sigma^i)_{\dot a}(S\sigma^j)_{\dot a}}{4\Pi^+}\right](z)\right]. \tag{3.21}$$

So the Lorentz algebra closes on on-shell states up to a gauge transformation.

The BRST operator $\widehat{Q}$ will now be related to the pure spinor BRST operator $Q = \int dz\lambda^\alpha d_\alpha$. The first step is to relate the cohomology of the BRST charge (3.16) to the cohomology of a charge $Q' = \int dz(\widehat{\lambda}_a\widehat{d}_a + \lambda_{\dot a}\widehat{d}_{\dot a})$ where $\lambda_{\dot a}$ has to satisfy $\lambda_{\dot a}\lambda_{\dot a} = 0$.

Suppose a state V is in the cohomology of Q', which implies that V is annihilated by

$$Q'' = \int dz(\widehat{\lambda}_a\widehat{d}_a + \widehat{\lambda}_{\dot a}\widehat{d}_{\dot a}) \tag{3.22}$$

up to terms proportional to $\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}$ or its derivatives. So

$$Q''V = \sum_{n=0}^{\infty}\partial^n(\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a})W_{(n)}\,, \tag{3.23}$$

for some $W_{(n)}$ for $n = 0$ to ∞. In addition, suppose that V has no poles with $\lambda_{\dot a}\lambda_{\dot a}$, i.e. V only depends on $w_{\dot a}$ in combinations which commute with the constraint on $\lambda_{\dot a}$. Then, from the relation

$$(Q'')^2 = \int dy \frac{(\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a})\widetilde{T}}{2}\,, \tag{3.24}$$

it follows that

$$Q^{''}W_{(n)} = \int dy \frac{1}{2}\widetilde{T}(y)V(z)\frac{(y-z)^n}{n!}\,. \tag{3.25}$$

Using the above equations, it is easy to check that the state $\widehat{V} = V + 2\sum_n(\partial^n c)W_{(n)}$ is annihilated by $\widehat{Q}$. Also, if a state V is BRST trivial up to terms $\partial^n(\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a})$, i.e.

$$V = Q^{''}\Omega + \sum_{n=0}^{\infty}\partial^n(\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a})Y_{(n)}\,, \tag{3.26}$$

for some $Y_{(n)}$, then $\widehat{V}$ is also BRST trivial with respect to $\widehat{Q}$ since

$$\widehat{V} = V + 2\sum_n(\partial^n c)W_{(n)} = \widehat{Q}(\Omega - 2\sum_n(\partial^n c)Y_{(n)})\,. \tag{3.27}$$

To complete the proof, one needs to show that any state in the cohomology of $\widehat{Q}$ can be mapped to a state in the cohomology of Q'. To show this, first note that any ghost-number one state $\widehat{V}$ in the cohomology of $\widehat{Q}$ with non-zero P^+ momentum can be expressed as $\widehat{V} = V + cW$ for some V and W which are independent of the (b,c) ghosts. This is because in light-cone gauge, the constraints $\widetilde{T}$ and $\widehat{d}_\alpha$ can be used to gauge away dependence on all variables except for X^j, S^a and the zero mode of X^+ in the integrated light-cone vertex operator $\int dz V_{LC}(X^j, S^a, X_0^+)$. So using the standard DDF construction, one can define a BRST-invariant vertex operator $\int dz V_{DDF}(X^j, S^a, X^+, \theta^\alpha)$ such that V_{DDF} coincides with V_{LC} when $\partial X^+ = \theta^\alpha = 0$. Since $\int dz V_{DDF}$ is BRST-invariant, $\widehat{Q}V_{DDF} = \partial\widehat{V}$ for some $\widehat{V}$. From the structure of the BRST operator $\widehat{Q}$ of (3.16), one learns that $\widehat{V} = \widehat{\lambda}^\alpha V_\alpha + cW$ where V_α and W are the double poles of $\widehat{d}_\alpha$ and $\widetilde{T}$ with V_{DDF}. Since $\partial(\widehat{Q}\widehat{V}) = \widehat{Q}\widehat{Q}V_{DDF} = 0$ and since there are no constant worldsheet fields, $\widehat{Q}\widehat{V} = 0$. Therefore, $\widehat{V} = \widehat{\lambda}^\alpha V_\alpha + cW$ is a ghost-number one vertex operator in the BRST cohomology which represents the light-cone state $\int dz V_{LC}$.

Since $\widehat{Q}(V + cW) = 0$ implies that $Q''V = \frac{1}{2}\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}W$, $Q'V = 0$ in the reduced Hilbert space. And $\widehat{V} = \widehat{Q}\Lambda$ where $\Lambda = \Omega + cY$ implies that $V = Q''\Omega - \frac{1}{2}\widehat{\lambda}_{\dot a}\widehat{\lambda}_{\dot a}Y$, i.e. $V = Q'\Omega$ in the reduced Hilbert space. So the cohomology of Q' is equivalent to the cohomology of $\widehat{Q}$ for states with non-zero P^+ momentum.[5]

To complete the equivalence proof, the relation between $Q^{'}$ and the pure spinor BRST operator $Q = \int dz \lambda^\alpha d_\alpha$ with $(\lambda\gamma^m\lambda) = 0$ has to be shown. For this purpose, it is convenient to define an antichiral spinor $r_{\dot a}$ satisfying $r_{\dot a}r_{\dot a} = 0$ and $r_{\dot a}\lambda_{\dot a} = 1$, and to split the fields S_a and $\widehat{\lambda}_a$ into[6]

$$\begin{aligned} S_a &= S_a^1 + S_a^2\,, \\ \widehat{\lambda}_a &= \widehat{\lambda}_a^1 + \widehat{\lambda}_a^2\,, \end{aligned} \tag{3.28}$$

[5] The equivalence proof does not hold for states of zero momentum since such states cannot be described by light-cone vertex operators. For example, the stress tensor $T = \{\widehat{Q}, \frac{1}{4}b\Pi^+ - \widehat{w}_\alpha\partial\theta^\alpha\}$ of (3.19) is BRST trivial, but the stress tensor in the pure spinor formalism is in the BRST cohomology since there are no states of negative ghost-number.

[6] As in the superparticle, $r_{\dot a}$ will only be allowed to appear in the combination $(\sigma^j r)_a(\sigma^j\lambda)_b$ so that the cohomology remains non-trivial.

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where

$$S^1_a = \frac{1}{2}(\sigma^j\lambda)_a(S\sigma^j r)\,,\ \ S^2_a = \frac{1}{2}(\sigma^j r)_a(S\sigma^j\lambda)\,,$$
$$\widehat{\lambda}^1_a = \frac{1}{2}(\sigma^j\lambda)_a(\widehat{\lambda}\sigma^j r)\,,\ \ \widehat{\lambda}^2_a = \frac{1}{2}(\sigma^j r)_a(\widehat{\lambda}\sigma^j\lambda)\,, \tag{3.29}$$

which have the OPE's:

$$\begin{aligned} S^1_a(y)S^2_b(z) &\longrightarrow \frac{(\sigma^i\lambda)_a(\sigma^i r)_b}{2(y-z)}\,, \\ S^1_a(y)S^1_b(z) &= S^2_a(y)S^2_b(z) \longrightarrow \text{regular}\,. \end{aligned} \tag{3.30}$$

In terms of these fields,

$$\begin{aligned} Q' &= \int dz(\lambda_{\dot a}\widehat{d}_{\dot a} + \widehat{\lambda}_a\widehat{d}_a) \\ &= \int dz(\lambda_{\dot a}d_{\dot a} - \frac{2\Pi^i(S^2\sigma^i\lambda)}{\sqrt{\Pi^+}} - 2\frac{:S^1_aS^2_a:\lambda_{\dot a}\partial\theta_{\dot a}}{\Pi^+} + \frac{(S^2\sigma^i\lambda)(S^2\sigma^i\partial\theta)}{\Pi^+} - \\ &\quad - 4\frac{\partial^2\theta_{\dot a}\lambda_{\dot a}}{\Pi^+} + 2\frac{\partial\Pi^+\partial\theta_{\dot a}\lambda_{\dot a}}{(\Pi^+)^2} + \widehat{\lambda}^1_a d_a + \widehat{\lambda}^2_a d_a + \\ &\quad + 2\widehat{\lambda}^1_a S^2_a\sqrt{\Pi^+} + 2\widehat{\lambda}^2_a S^1_a\sqrt{\Pi^+})\,, \end{aligned} \tag{3.31}$$

where :: denotes normal ordering, i.e. $:S^1_aS^2_b(z):\equiv \int dyS^1_a(y)S^2_b(z)(y-z)^{-1}$. Performing the similarity transformation $Q' \to e^{-\int dzA}Q'e^{\int dzA}$ where $A = \frac{d_aS^2_a}{2\sqrt{\Pi^+}}$, one obtains

$$\begin{aligned} Q' = \int dz\Big(&\lambda_{\dot a}d_{\dot a} - \frac{2:S^1_aS^2_a:\lambda_{\dot a}\partial\theta_{\dot a}}{\Pi^+} + \frac{4(\partial\theta_{\dot a}\lambda_{\dot a})(\partial\lambda_{\dot b}r_{\dot b})}{\Pi^+} - \frac{2\partial\Pi^+\partial\theta_{\dot a}\lambda_{\dot a}}{(\Pi^+)^2} + \\ &+ \widehat{\lambda}^1_a d_a + 2\widehat{\lambda}^2_a S^1_a\sqrt{\Pi^+}\Big)\,. \end{aligned} \tag{3.32}$$

To simplify further the BRST charge, one performs the additional similarity transformation $Q' \to e^{-\int dzB}Q'e^{\int dzB}$ where

$$B = -\frac{\partial\Pi^+}{2\Pi^+} + \frac{1}{2}:S^1_aS^2_a:\log\Pi^+ + \frac{4(\partial\theta_{\dot a}\lambda_{\dot a})(\partial\theta_{\dot b}r_{\dot b})}{\Pi^+}\,, \tag{3.33}$$

to obtain

$$Q' = \int dz(\lambda_{\dot a}d_{\dot a} + \widehat{\lambda}^1_a d_a + 2\widehat{\lambda}^2_a S^1_a)\,. \tag{3.34}$$

So after performing these similarity transformations, $Q' = \int dz(\lambda^\alpha d_\alpha + 2\widehat{\lambda}^2_a S^1_a)$ where λ^α is a pure spinor defined by

$$[\lambda_{\dot a}, \lambda_a] = [\lambda_{\dot a}, \widehat{\lambda}^1_a]\,. \tag{3.35}$$

Making use of the standard quartet argument, the cohomology of $Q' = Q + 2\int dz\widehat{\lambda}^2_a S^1_a$ is equivalent to the cohomology of $Q = \int dz\lambda^\alpha d_\alpha$ in a Hilbert space independent of S^1_a, $\widehat{\lambda}^2_a$ and their conjugate momenta S^2_a and $\widehat{w}^1_a$. So, it has been shown that the Green-Schwarz superstring action and BRST operator in semi-light-cone gauge are equivalent to the action

$$S = \frac{1}{\pi}\int d^2z\left[\frac{1}{2}\partial X^m\overline{\partial}X_m + p_\alpha\overline{\partial}\theta^\alpha + w_\alpha\overline{\partial}\lambda^\alpha + \text{anti-holomorphic terms}\right], \tag{3.36}$$

and BRST operator $Q = \int dz\lambda^\alpha d_\alpha$ where λ^α is a pure spinor.

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